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SOLUTION OF INTEGER LINEAR PROGRAMMING PROBLEMS WITH TRIANGULAR FUZZY NUMBERS

Someshwar Siddi*, Dr. Y. Raghunatha Reddy
* Research Scholar, Department of OR & SQC, Rayalaseema University, India
Assistant Professor, Department of OR&SQC, Rayalaseema University, India

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ABSTRACT
Fuzzy Integer Linear Programming problem is an application of fuzzy set theory in linear decision problems and most of these problems are related to linear programming with fuzzy variables. In this paper, we proposed a method for Integer linear programming problems with fuzzy variables. Two numerical examples were illustrated with the help of the proposed method. This method is a simple tool for the best solution to a variety of Integer linear programming problems.

KEYWORDS: Fuzzy linear programming problems, Integer linear programming, Triangular fuzzy numbers.

INTRODUCTION
Linear Programming (LP) is one of the most important Operational Research (OR) techniques in real world problems. LP requires much well-defined and precise data which involves high information costs. Furthermore the optimal solution of an LP only depends on a limited number of constraints and thus much of the information collected has little impact on the solution.

In order to reduce information costs and at the same time avoid unrealistic modeling, the use of fuzzy linear programming has been introduced. Fuzzy Linear Programs has been applied to many disciplines such as analysis of water use in agriculture, farm structure optimization problem, assignment problems, capital asset pricing model, optimal allocation of production of metal in manufacturing, production-mix selection problem, air pollution regulation problem, coordination of personnel demand and available structure, transportation problem and many more industrial applications.

There are certain decision problems where decision variables make sense only if they have integer values in the solution. for example, it does not make sense saying 1.5men working on a project or 1.6 machines in a workshop. Capital budgeting, construction scheduling, plant location and size, routing and shipping schedule, batch size, capacity expansion, fixed charge, etc., are few problems which demonstrate the areas of application of Integer Programming.

Maleki et al. [1] discussed Linear Programming problems with Fuzzy Variables. **Mjelde [2]** discussed Fuzzy resource allocation. **Ganesan et al. [3]** solved many Fuzzy Linear Problems with Trapezoidal Fuzzy Numbers. **Mahmoud et al. [4]** presented a survey and some applications of multiple objective (fuzzy) dynamic programming problems. **Vasant et al. [5]** used fuzzy linear programming technique in Decision making in industrial production planning. **Bellman et al. [6]** set the basic principles of decision making in fuzzy environments, which have been used as a building blocks of fuzzy linear programming. **Allahviranloo et al. [7]** discussed Fuzzy integer linear programming problems. **Takashi [8]** solved Fuzzy linear programming problems as bi-criteria optimization problem. **Zimmermann [9]** extended his fuzzy linear programming approach to several objective functions. We considered fuzzy integer linear programming problems involving triangular fuzzy numbers and introduced the fuzzy optimal solution for these problems.

PRELIMINARIES

Definition 1: A subset A of a set X is said to be fuzzy set if $\mu_A : X \rightarrow [0,1]$, where μ_A denote the degree of belongingness of A in X.

Definition 2: The support of a fuzzy set A on R is the crisp set of all xεR such that $\mu_A(x) > 0$.

Definition 3: Let (a,b,c) and (d,e,f) be two triangular fuzzy numbers, then

[1] (a,b,c)+(d,e,f)=(a+d,b+e,c+f).
[2] k(a,b,c)=(ka,kb,kc) for k≥0.
[3] k(a,b,c)=(kc,kb,ka) for k<0.

Definition 4: A non negative fuzzy vector $\tilde{x}$ is said to be the solution of the fuzzy linear system $A\tilde{x} \leq \tilde{b}$ if $\tilde{x}$ satisfies $A\tilde{x} \leq \tilde{b}$.

FUZZY INTEGER LINEAR PROGRAMMING

Consider the following integer linear programming with fuzzy variables:

(P) Maximize: $\tilde{z}=c\tilde{x}$
subject to $A\tilde{x} \leq \tilde{b}$,
$\tilde{x} \geq 0$ are integers,

where $A=(a_{ij})_{mxn}$ is a nonnegative real crisp matrix, c is nonnegative crisp vector, $\tilde{x} =(\tilde{x}_j)_{nx1}$ and $\tilde{b} =(\tilde{b}_i)_{mx1}$ are non negative real fuzzy vectors for all 1≤j≤n and 1≤i≤m.

Definition 5: A fuzzy vector $\tilde{x}$ is said to be a feasible solution of the problem (P) if $\tilde{x}$ satisfies $A\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$.

Definition 6: A fuzzy vector $\tilde{x}$= (x1,x2,x3) is an optimal solution of the problem iff x_1,x_2,x_3 are optimal solutions of the following crisp integer linear programming problems respectively

(P1) Maximize: $z_1=cx_1$
subject to $Ax_1\leq b_1$,
$x_1\geq 0$ are integers

(P2) Maximize: $z_2=cx_2$
subject to $Ax_2\leq b_2$,
$x_2\geq 0$ are integers

(P3) Maximize: $z_3=cx_3$
subject to $Ax_3\leq b_3$,
$x_3\geq 0$ are integers.

Numerical Examples

The proposed method is illustrated by the following examples.

Example 1. Maximize $Z = x_1+x_2$
subject to $3x_1+ 2x_2 \leq (5,10,15)$
$x_2\leq (2,5,9)$
x1, x2≥ 0.

Let $\tilde{z} = (z_1,z_2,z_3)$, $\widetilde{x_1} = (x_1,y_1,u_1)$ and $\widetilde{x_2} = (x_2,y_2,u_2)$.

Now the fuzzy integer linear programming problem is converted into crisp integer linear programming problem is given below:

(P1) Maximize $z_1 = x_1+x_2$
subject to $3x_1+ 2x_2 \leq 5$
$x_2\leq 2$
x1, x2≥ 0 are integers.

(P2) Maximize $z_2 = y_1+y_2$
subject to $3y_1+ 2y_2 \leq 10$
$y_2\leq 5$
y1, y2≥ 0 are integers.

(P3) Maximize $z_3 = u_1+u_2$
subject to $3u_1+2 u_2 \leq 15$
$u_2\leq 9$
u1, u2≥ 0 are integers.

Now, using an algorithm for Integer Linear Programming problem,

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The non integer solution of the problem (P1) is x_1=0.3333, x_2=2 and z_1=2.3333.
The integer solution of the problem (P1) is x_1=1, x_2=1 and z_1=2.
The non integer solution of the problem (P2) is y_1=0, y_2=5 and z_2=5.
The integer solution of the problem (P2) is y_1=0, y_2=5 and z_2=5.
The non integer solution of the problem (P3) is u_1=0, u_2=7.5 and z_3=7.5.
The integer solution of the problem (P3) is u_1=1, u_2=6 and z_3=7.
Therefore, the integer solution for the given fuzzy integer linear programming problem is
$\widetilde{x_1} = (x_1,y_1,u_1) = (1,0,1)$, $\widetilde{x_2} = (x_2,y_2,u_2) = (1,5,6)$ and $\tilde{z} = (z_1,z_2,z_3)=(2,5,7)$.

Example 2. Maximize $Z = 4x_1+6x_2$
subject to $x_1+ 2x_2 \leq (4,7,14)$
$2x_1+3x_2 \leq (5,10,14)$
$x1, x2 \geq 0$.

Let $\tilde{z} = (z_1,z_2,z_3)$, $\widetilde{x_1} = (x_1,y_1,u_1)$ and $\widetilde{x_2} = (x_2,y_2,u_2)$.
Now the fuzzy integer linear programming problem is converted into crisp integer linear programming problem is given below:

(P1) Maximize $z_1 = 4x_1+6x_2$
subject to $x_1+2x_2 \leq 4$
$2x_1+3x_2 \leq 5$
$x1, x2 \geq 0$ are integers.

(P2) Maximize $z_2 = 4y_1+6y_2$
subject to $y_1+2\ y_2 \leq 7$
$2y_1+3y_2 \leq 10$
$y1, y2 \geq 0$ are integers.

(P3) Maximize $z_3 = 4u_1+6u_2$
subject to $u_1+ 2u_2 \leq 14$
$2u_1+3u_2 \leq 14$
$u1, u2 \geq 0$ are integers.

Now, using an algorithm for Integer Linear Programming problem,
The non integer solution of the problem (P1) is x_1=0, x_2=1.6667 and z_1=10.
The integer solution of the problem (P1) is x_1=1, x_2=1 and z_1=10.
The non integer solution of the problem (P2) is y_1=0, y_2=3.3333 and z_2=20.
The integer solution of the problem (P2) is y_1=2, y_2=2 and z_2=20.
The non integer solution of the problem (P3) is u_1=0, u_2=4.6666 and z_3=28.
The integer solution of the problem (P3) is u_1=1, u_2=4 and z_3=28.

Therefore, the solution for the given fuzzy integer linear programming problem is
$\widetilde{x_1} = (x_1,y_1,u_1) = (1,2,1)$, $\widetilde{x_2} = (x_2,y_2,u_2) = (1,2,4)$ and $\tilde{z} = (z_1,z_2,z_3)=(10,20,28)$.

CONCLUSION

The proposed method provides a solution to fuzzy integer linear programming problems. This method is a simple tool for the best solution to a variety of Integer linear programming problems with fuzzy variables in a simple and effective manner.

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